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RISK ANALYSIS AND PROBABILITY OF FAILURE OF A GRAVITY DAM *

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1. INTRODUCTION

Risk analysis techniques as applied to dams demand the evaluation of probabilities of failure for different failure modes. An application example is presented herein for the spillway section of a spanish concrete gravity dam. Probability of failure against sliding along the dam-foundation contact plane is estimated for different given water levels. As each water level has its own annual probability ("event probability"), the calculated probability of failure is "conditional probability". Sliding is modelled with a simple limit equilibrium model. Probability of failure is estimated with Level 2 and Level 3 reliability methods. Sensitivity of probabilities to rock-to-concrete tensile strength is evaluated.

* *Analyse des risques et probabilité de rupture d'un barrage-poids*

2. PROBABILITY OF SLIDING FAILURE OF A CONCRETE GRAVITY DAM

2.1. CASE OF STUDY

The spillway section of a spanish concrete gravity dam is considered. Top of the dam is at 82.50 m over the dam-foundation contact. Spillway crest level is 73 m. The spillway is equiped with Taintor gates allowing a Maximum Normal Water Level (M.N.W.L.) of 80 m. The probability of failure will be estimated for 8 upstream levels: 60 , 73, 75, 80 (M.N.W.L.), 82.50 (top), 84 (1.50 m overtopping), 86 (3.50 m overtopping) and 90 m (7.50 m overtopping). These high water levels call for extremely low-probability hidrological events (Ref [1]).

2.2. LIMIT EQUILIBRIUM MODEL

Sliding stability along the dam-foundation contact plane can be analysed by means of a simple two-dimensional limit equilibrium model. Hydrostatic load and sediment pressure are the driving forces, S. Shear strength, R, is calculated with (1),

$$R = (N - U)tg\varphi + B \times c \qquad \text{(Eq. 1)}$$

where:

N (N/m) is the sum of vertical forces acting on the dam-foundation contact.
U (N/m) is the uplift.
B (m^2/m) is the area in compression in the dam-foundation contact.
φ (º) is the friction angle in the contact.
c (Pa) is the cohesion in the contact.

A performance function g* is defined as:

$$g^* = \frac{R}{S} - 1 \qquad \text{(Eq. 2)}$$

The model considers the uplift distribution according to drain location and drain effectiveness as proposed by Corps practice (Ref. [2]).

The model is implemented in a spreadsheet an accounts for crack opening and crack propagation, updating uplift pressure assuming 100% of uplift acting along the cracked length. Criteria for crack opening and crack propagation is taken from the Corps (Ref. [2]). A crack initiates when tensile stress acting along the contact plane exceeds rock-to-concrete tensile strength.

2.3. RANDOM VARIABLES CONSIDERED.

On Table 1 a list of random variables for static equilibrium analysis is presented. There are other variables that have been considered in the analysis with constant values, assuming that they are subjected to very low uncertainty. It is the case, for instance, of the dam cross section area. Values have been assigned based on dam safety review documents and, when not available, on published data. The use of unbounded probability density functions may be deemed of not having physical meaning as in the real world parameters are certainly bounded. Care should be taken when using this unbounded probabilility functions, assuring that most of the mass probability (say 99.999%) concentrates on the interval of values that “makes sense” according to experimental and/or published data. Another strategy is to truncate the distributions between a minumum and maximum values, assigning a probability of 1 to the new bounded interval. Corrections should be made to distribution parameters to account for this truncation. It should be noted that compressive strength of concrete and rock mass foundation are considered in the model only in the case when a crack develops along dam-foundation to an extent such that the compressive stress generated in the toe exceeds the compressive strength of the materials. Such situation is considered as a failure. According to this, only one of the variables (the one corresponding to a less strength) is needed.

2.4. PROBABILITY OF FAILURE WITH LEVEL 2 METHODS.

The conditional probability of failure by sliding along the dam-foundation contact can be assessed using Level 2 methods of reliability analysis. Level 2 or “Second Moment Methods” use only the first two moments (mean and standard deviation) of the probability distribution of the random variables. The Taylor’s series expansion of the performance function g* is among these methods. One of the advantages of the method is that provides a measure of the contribution of each individual random variable to the variance of the performance function, helping in the selection of the relevant random variables. So it is a very useful tool for filtering variables and selecting the most important ones that may go into further analyses with more complex and accurate methods.

The Taylor’s series method has been used to analyse the contribution of each of the aforemention random variables to the variance of the performance function. The results are presented in Figure 2. Only two variables, rock-to-concrete friction angle and cohesion, explain most part of the variance of the performance function. According to this, it is justified to consider these two variables as the only random variables while the rest can be considered in the analysis with constant, fixed values (Ref. [3]). Probabilities of failure obtained with this method are depicted in Figure 3 for different upstream water levels.

Table 1
Random variables for static limit equilibrium analysis Tableau 1.
Variables aléatoires pour l'analyse de l'équilibre limite statique.

Variable	**Unit**	**PDF**	**Mean**	**SD**	**Min**	**Max**
Concrete mass density ((c)	kg/m3	Normal	2350	50	2150	2550
Drainage inefficiency (K)	-	Triangular	0.37	0.22	0.00	1.00
Uplift coefficient over Maximum Normal W.L.(α)	-		0.50		0.00	1.00
Sediments Specific weight ((sed)	N/m3	Triangular	873g	99g	650g	1130g
Sediments height (Hsed)	m	Uniform	7.75	4.47	0.00	15.50
Sediment pressure coefficient (K_{sed})	-	Triangular	0.415	0.035	0.333	0.500
Concrete compressive strength ($\sigma_{c,c}$)	MPa	Normal	13.00	1.30	-∞	+∞
Rock Mass Compressive strength (σ_{cm})	MPa	Normal	9.06	0.60	-∞	+∞
Dam-foundation contact Friction angle (φ)	º	Normal	50	10	30	70
Dam-foundation contact Cohesión (c)	MPa	Lognormal	0.40	0.30	0.00	2.00
Rock-to-concreteTensile strength ($\sigma_{i,t}$)	MPa	Normal	0.90	0.22	0.00	1.80

2.5. PROBABILITY OF FAILURE WITH LEVEL 3 METHODS.

A more rigurous approach can be done in terms of Level III methods of analysis, by means of the Monte Carlo method. The limit equilibrium model has been implemented on a spreadsheet and the statistical analysis has been carried out with the tool @RISK (Palisade). Sampling of the random variables is done in a fashion consistent with the probability density functions of the variables, so all the statistical information is transferred into the calculations, and not only the first two moments of the distributions.

Monte Carlo analysis have been done in two scenarios: (a) with only two random variables – friction and cohesion in the rock-to-concrete contact - and (b) with the full set of ten variables considered as random variables. Results show a relatively small difference between the predicted probabilities in both scenarios, with slightly larger probabilities of failure in case (b).

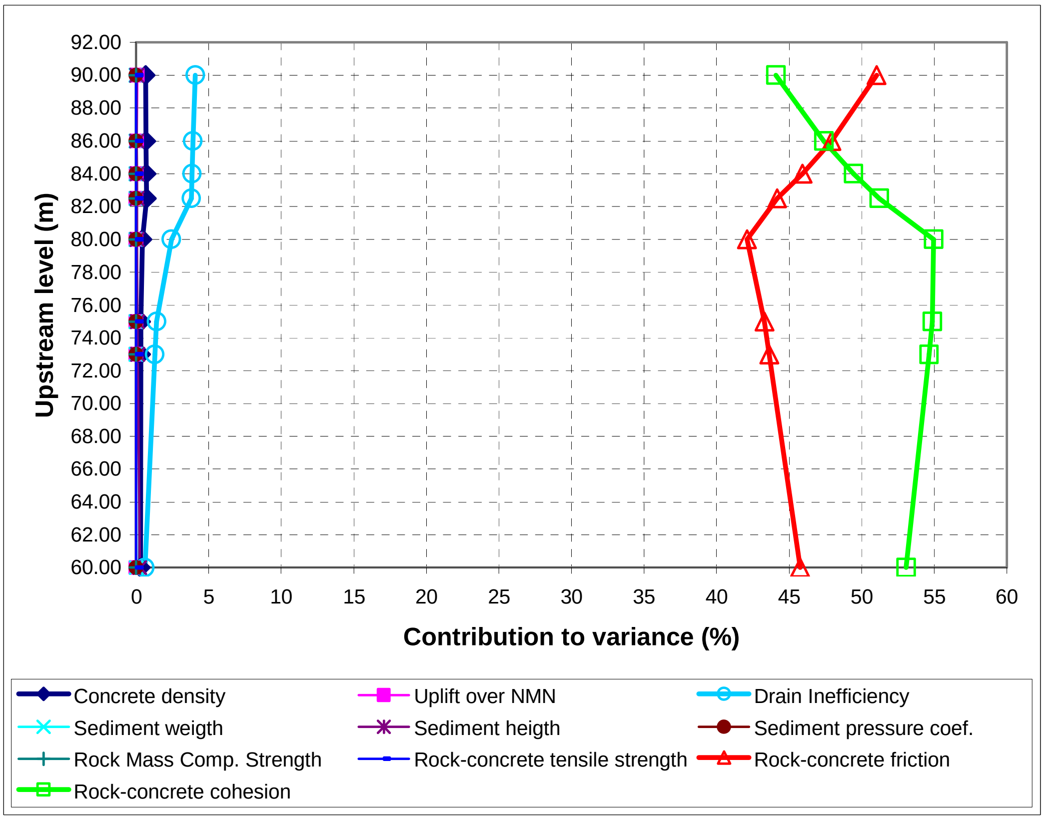

Fig. 2

Contribution of random variables to variance of the performance function

Contribution des variables aléatoires à l'écart de la fonction de performance.

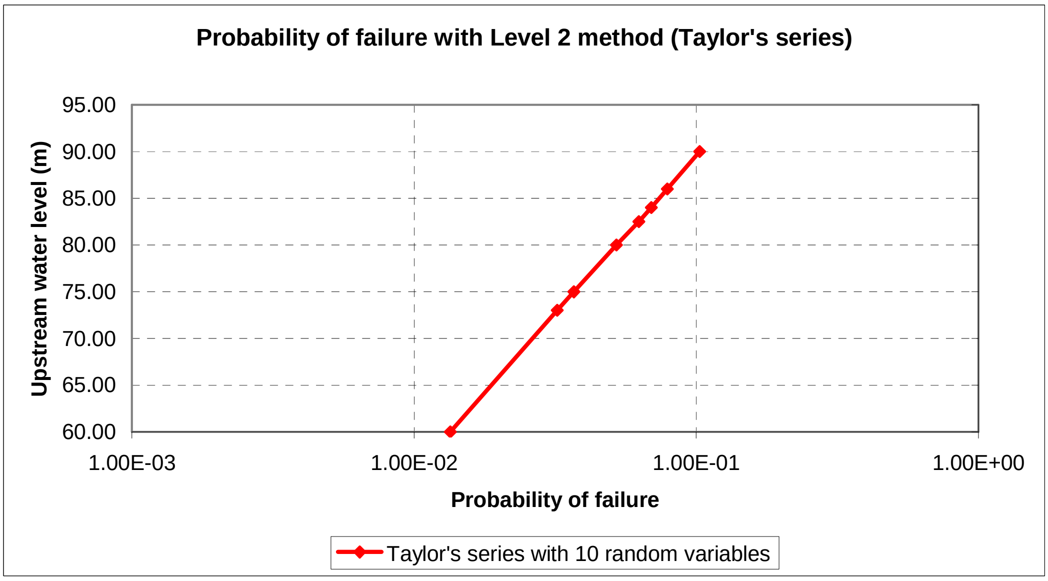

Fig. 3

Probabilities of failure with a level 2 method

Probabilité de rupture avec la méthode de niveau 2

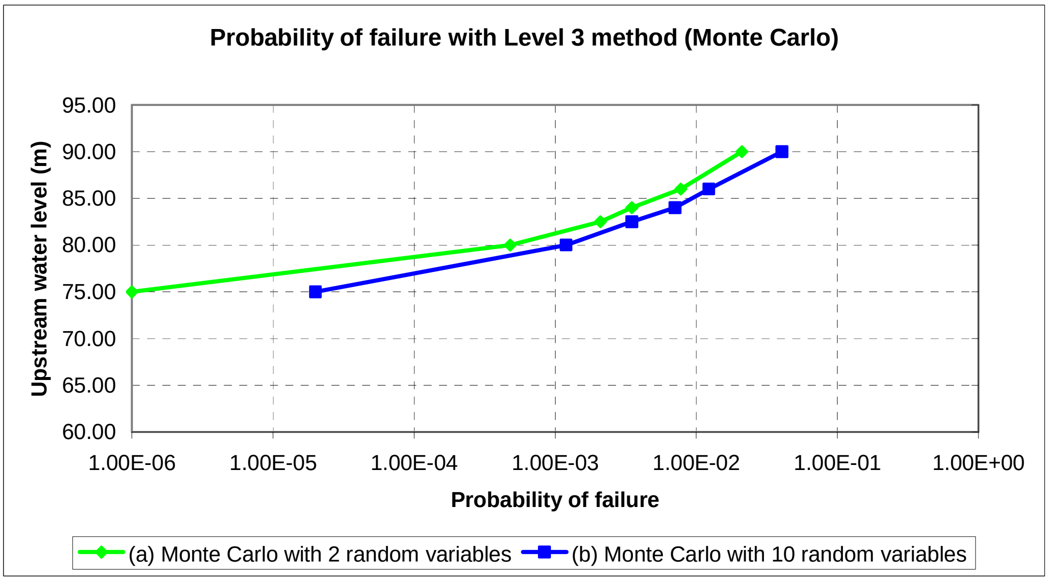

Fig. 4

Probabilities of failure with level 3 (Monte Carlo) method

Probabilité de rupture avec la méthode de niveau 3 (Monte Carlo)

Comparison can be made between the probabilities predicted for scenario (b) with level 2 and level 3 methods. Results show that probabilities of failure can be overestimated unrealistically with level 2 methods by more than an order of magnitude.

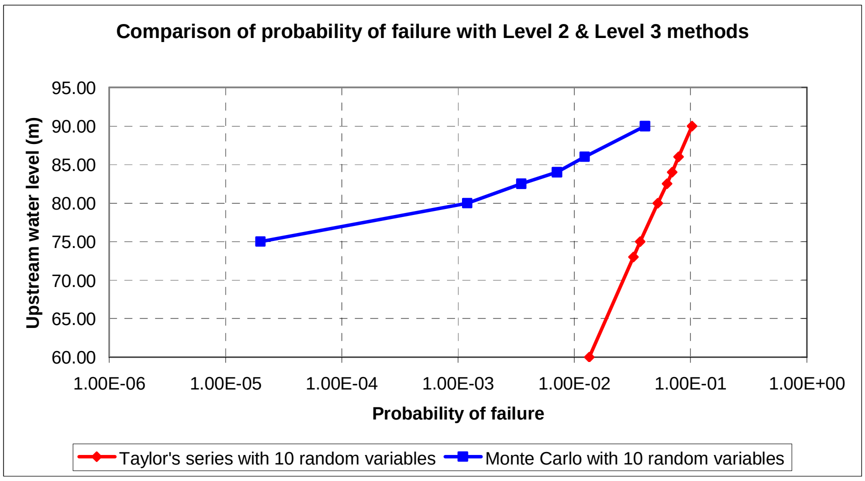

Fig. 5

Comparison of probability of failure with level 2 and level 3 methods

Comparaison de probabilité de rupture entre les méthodes de niveau 2 et 3

3. UNCERTAINTIES IN STRESSES IN THE CONTACT PLANE.

3.1. SOURCES OF UNCERTAINTY

Uncertainties in stresses in rock-to-concrete contact plane are related to: (a) uncertainties in the rock-to-concrete tensile strength, and (b) uncertainties in the stress distribution acting along the contact plane. If the rock-to-concrete tensile strength adopted according to published data (Ref. [4]) is higher than the tensile stress calculated by the limit equilibrium model, a crack never develops, leading to dam stability. If the tensile strength is set to values provided by probe testing, the size effect law (the negative correlation between size of the model and its cracking strength) is being ignored. Limit equilibrium models have been employed in engineering practice with very low or null values of the rock-to-concrete tensile strength, and so taken into account somehow the size effect law. Limit equilibrium model assumes linear distribution of stressess ignoring the peakedness of its distribution near the crack tip.

3.2. INCREASE OF THE COEFFICIENT OF VARIATION

Analysis can be carried out increasing the coefficient of variation (ratio of the standard deviation to the mean) of the probability density function of the rock-to-concrete tensile strength, to check the variations in failure probability estimation. If C.O.V. is increased from 0.24 to 0.50, the new probability distribution for the tensile strength remains as listed in Table 2.

Table 2
Rock-to-concrete probability distribution with C.O.V. = 0.5
Répartition de probabilité roche-béton avec C.O.V. = 0,5

Variable	**Unit**	**PDF**	**Mean**	**SD**	**Min**	**Max**
Rock-to-concrete Tensile strength ($\sigma_{i,t}$)	MPa	Normal	0.90	0.45	0.00	1.80

Results for Level 2 analysis exhibit no difference with the previous results, as the performance function values evaluated do not vary. This illustrates the difficulties of Level 2 methods to capture non-linear effects. Results for Level 3 analysis show significant differences in probabilities of failure.

3.3. REDUCTION OF ROCK-TO-CONCRETE TENSILE STRENGTH

Let's suppose that the tensile strength is reduced to a 50% of its original value. The new probability density function remains as listed in Table 3.

Table 3
Rock-to-concrete probability distribution with 50% reduction
Répartition de probabilité roche-béton avec une réduction de 50 %

Variable	Unit	PDF	Mean	SD	Min	Max
Dam-foundation contact Tensile strength ($\sigma_{i,t}$)	MPa	Normal	0.45	0.11	0.00	0.90

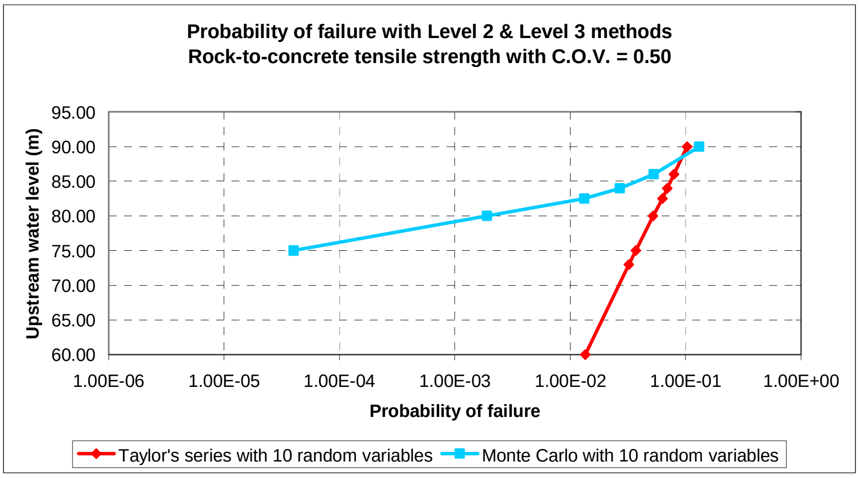

Fig. 6
Probability of failure with level 2 and level 3 methods (C.O.V. = 0.5)
Probabilité de rupture avec les méthodes de niveau 2 et 3 (C.O.V. = 0,5)

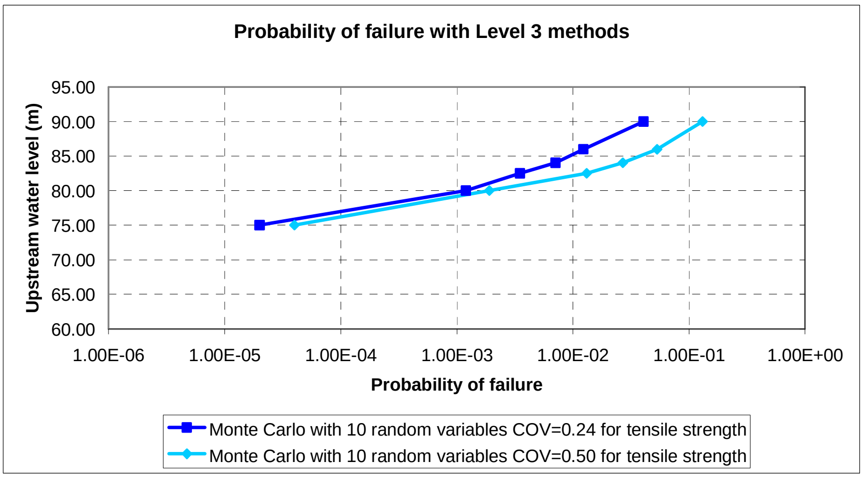

Fig. 7
Probability of failure with level 3 method (C.O.V. = 0.24 and C.O.V. = 0.5)
Probabilité de rupture avec la méthode de niveau 3 (C.O.V. = 0,24 et C.O.V. = 0,5)

Results obtained for 50% reduction with Level 2 methods exhibit the same probabilities of failure for all upstream levels, except for 90 m, where equilibrium is not achieved, as a crack propagates downstream without stabilisation and exceeding rock mass compressive strength at the toe. A performance function cannot be evaluated for such level. On the other hand, with Level 3 method a significant increase of the estimated probability is obtained due to reduction of rock-to-concrete tensile strength, above all for higher water levels.

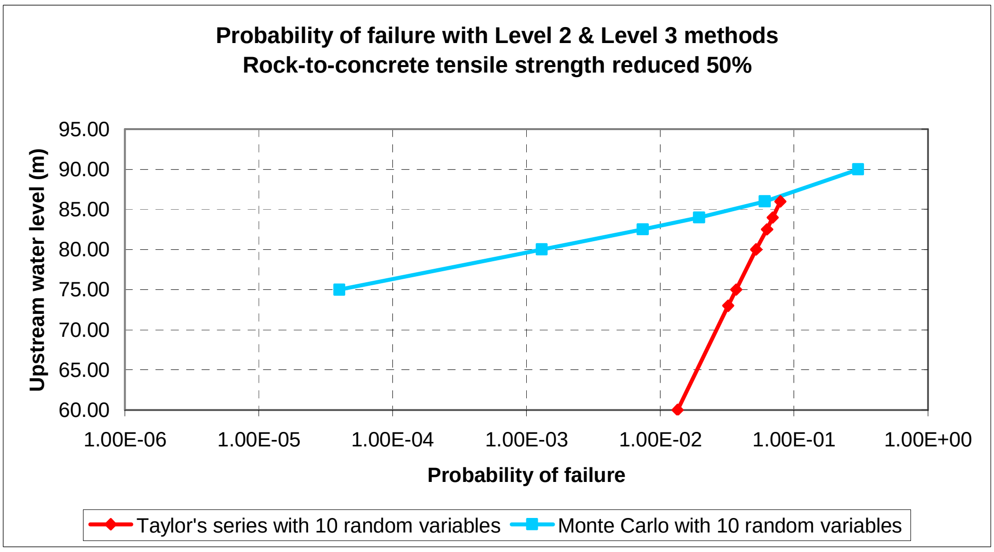

Fig. 8

Probability of failure with level 2 & 3 methods (tensile strength reduction of 50%)

Probabilité de rupture avec les méthodes de niveau 2 et 3 (réduction de résistance à la traction de 50 %)

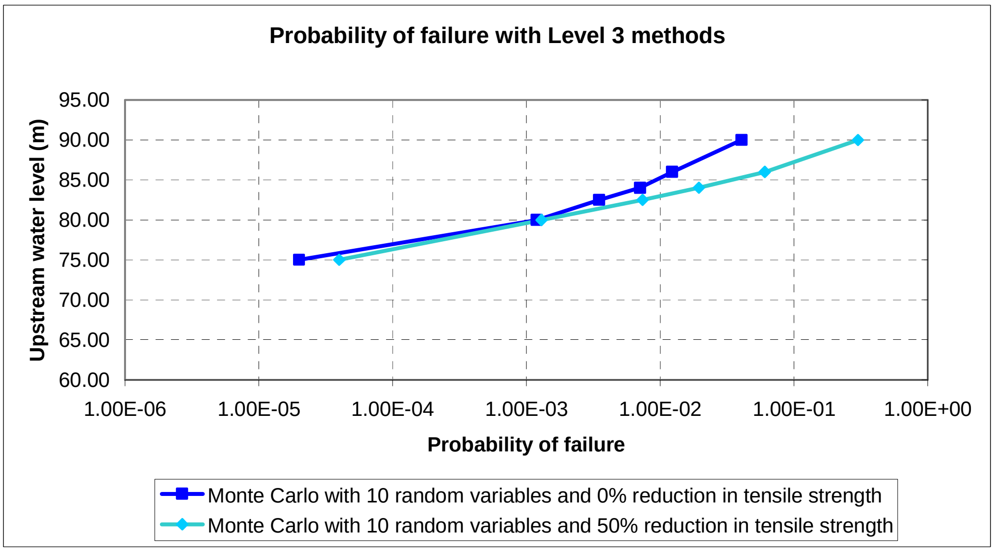

Fig. 9

Probability of failure with level 3 method (tensile strength reduction of 50%)

Probabilité de rupture avec la méthode de niveau 3 (réduction de résistance à la traction de 50 %)

3.4. UNDERESTIMATION OF STRESS LEVELS PREDICTED

Due to its on formulation, limit equilibrium models underestimate the tensile stress levels acting on the heel of the contact plane between dam and foundation. As the hypothesis on tensile strength values in normal practice are strongly conservative, this underestimation is offset somehow.

In the risk analysis environment, limit equilibrium models offer a useful tool as statitiscal realibility-based methods match very well with them (performance functions are formulated and evaluated easily). As knowledge on parameters influencing the behaviour of the dam increases, the use of more complex models becomes justified (Ref. [5]). In fact, outside risk analysis, a strong degree of complexity has been achieved in last decades involving numerical models (finite element and finite difference with large number of elements), fracture mechanics, fully non linear dynamic analysis, and so on.

As risk analysis techniques evolve in time, become more familiar to users, and their results are applied by dam owners, the need to step beyond the simple limit equilibrium methods for better estimation of failure probabilities will arise. In particular, problems related to cracking on dam-foundation contact plane, can be faced with relatively simple tension-based criteria or with more complex fracture mechanics concepts. Better estimation of stress levels acting on the base of the dam can be achieved with models based on a deformable body approach, starting with the simplest linear elastic constitutive model for dam and foundation in combination with the well-known Mohr-Coulomb model for the interface. The ratio between modulus of elaticity of dam and its foundation plays a key role in the stress levels acting on the tip of a possible crack.

Table 4
Summary of results
Résumé des résultats

Reliability method	**Probability of failure**		**Ableness to capture non-linearities**
	Low upstream water levels	**High upstream water levels**	
Level 2	Over-estimated	May be under-estimated	Low
Level 3	Realistic	Realistic	High

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REFERENCES

[1] MEMBRILLERA, M.; ESCUDER, I.; GONZALEZ, J.; ALTAREJOS, L.: Aplicacion del Analisis de Riesgos a la Seguridad de Presas. I.S.B.N.: 84-9705-779-1. 2005. Pages: 105. Ed: Universidad Politecnica de Valencia. Valencia, 2005.

[2] US ARMY CORPS OF ENGINEERS. Evaluation and comparison of stability analysis and uplift criteria for concrete gravity dams by three federal agencies. ERDC/ITL TR-00-1. Washington, DC., 2000.

[3] OOSTHUIZEN, C. : Assessing uncertainties during dam safety evaluations : a South African perspective. 73rd Annual Meeting of ICOLD. Tehran, IRAN. May 1-6, 2005. Paper nº :106-S1.

[4] RUGGERI, G. : Uplift pressures under concrete dams. Final report. ICOLD European Club. Working group on uplift pressures under concrete dams. 2004.

[5] WIELAND, M. ; GUIMOND, R. ; MALLA, S. :Uncertainties in seismic analysis of concrete and embankment dams. 73rd Annual Meeting of ICOLD. Tehran, IRAN. May 1-6, 2005. Paper nº :015-S1

SUMMARY

Estimation of the probability of failure against sliding of the spillway section of a spanish gravity dam is carried out as a part of the dam risk analysis. The behaviour is modelled with a simple limit equilibrium model. Probabilities of failure are estimated with reliability-based Level 2 (Taylor’s series) and Level 3 (Monte Carlo) methods. Results show that Level 2 methods are less sensitive to variations related with non linear effects, predicting unrealistically high probabilities of failure for lower upstream water levels. Influence of rock-to-concrete tensile strength is analised, and sensitivity of the probabilities to this parameter is checked. The use of more elaborated models in risk analysis arises as a need for further developments in this field.

RÉSUMÉ

L'estimation de la probabilité de rupture due au glissement de la partie déversante d'un barrage-poids espagnol est réalisée dans le cadre d'une l'analyse de risques du barrage. Le comportement du barrage est reproduit à l'aide d'un modèle d'équilibre limite. Les probabilités de rupture sont estimées grâce aux méthodes fiabilistes de niveau 2 (séries de Taylor) et de niveau 3 (Monte Carlo). Les résultats montrent que les méthodes de niveau 2 sont moins sensibles aux variations liées à des effets non linéaires et ont pour résultats des probabilités fortement irréalistes de rupture pour des niveaux d'eau en amont faibles. L'influence de la résistance à la traction d'une structure roche-béton est analysée, et la sensibilité des probabilités face à ce paramètre est vérifiée. L'utilisation de modèles plus élaborés dans le cadre de l'analyse de risques constitue une nécessité pour que des progrès soient réalisés dans ce domaine.